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Robot Formation Control in Leader-Follower Motion Using Direct Lyapunov Method

Xiaohai LI and Jizhong XIAO

Abstract — **In this paper, we investigate the leader-following based formation control of nonholonomic mobile robots. We present a new kinematics model for the leader-follower system using Cartesian coordinates. Based on this new model and the direct Lyapunov method, a globally stable controller is derived for the whole system. Simulation results are included to verify the feasibility of the proposed model and controller.**

Index Terms – Formation control, nonholonomic mobile robots, direct Lyapunov method.

1. INTRODUCTION

Coordination and control of distributed robots drew extensive research interest in the robotics and control communities during the last few years. As one of the research topics, the formation control of multiple robots is studied extensively with applications in mobile robots, unmanned aerial vehicles (UAVs), autonomous underwater vehicles (AUVs), aircraft, and satellites [12, 11]. A variety of strategies and approaches have been proposed for formation control in the literature. They can be roughly categorized as virtual structure, behavior based, and leader-following, to name a few.

The virtual structure approach treats the entire formation as a single rigid entity [10, 12]. Desired motion is assigned to the virtual structure as a whole; and the trajectories for each robot to follow are traced out. The virtual structure can evolve as a whole in any direction with some given orientations and maintaining a rigid geometric relationship among group members. The main disadvantage of the current virtual structure implementation is centralization, which leads to a single point of failure for the whole system.

The basic idea of the behavior-based approach is to prescribe several desired behaviors for each robot, and then the final action of each robot is derived by weighting the relative importance of each behavior. Possible behaviors include, for instance, goal seeking, obstacle avoidance, collision avoidance and formation keeping [1, 15]. The limitation of the behavior-based approach is that it is difficult to analyze the system's performance mathematically. Thus, it is hard to guarantee a precise formation control.

In the leader-following approach [4, 5, 6, 13], one or a few robots are designated as the leader, with the rest being followers. The follower robots need to position themselves relative to the leader and maintain a desired relative position with respect to the leader.

A few models and controllers have been presented in the literature [4, 5, 6, 13] which primarily use a polar coordinate based representation. However, the polar coordinate representation has a potential singularity problem, i.e., the denominator of the derived controller may be zero at some time instant. A controller by input-output linearization is presented in [4, 5], but it is only stable locally. A globally stable but complicated sliding mode controller is presented in [13].

By the leader-following approach, to prescribe a formation maneuver we only need to specify the leader's trajectory and the desired relative positions and orientations between leaders and followers. When the motion of the leader is known, the desired positions (distance and heading angle) of the followers relative to the leader can be achieved by local control of each follower. Therefore, in a certain sense, the formation control problem can be essentially viewed as an extension of the traditional trajectory tracking problem. Based on this observation, we utilize some common techniques of trajectory tracking control of nonholonomic mobile robots [3, 7, 8] in this paper, and we derive a new model and controller for the leader-following based formation control of mobile robots.

As we know, the representation of the kinematics model of mobile robots for trajectory tracking can use either Cartesian or polar coordinates. Although the distance and orientation of a robot can be easily represented by polar coordinates, Cartesian coordinate representation can avoid the possible singular points of polar coordinate representation. In this paper we exploit Cartesian coordinate presentation and derive a new kinematics model for the leader-follower configuration of nonholonomic mobile robots. Based on this new model, we use the Lyapunov direct method to derive a globally, not just locally, stable controller for the whole system.

This paper is organized as follows. In Section II, we derive a new kinematics model using Cartesian coordinates for the formation control of two nonholonomic mobile robots. In Section III, a globally stable controller is designed using the direct Lyapunov method, and the stability of the whole system is also analyzed. A few

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simulation results are included in Section IV to verify the feasibility of the new model and controller. Finally, this paper ends with conclusions and future work in Section V.

2. SYSTEM MODELING

In this section, we derive a new kinematics model using Cartesian coordinates for the leader-following based formation control of a team of two nonholonomic mobile robots. For simplicity, we consider the configuration of a team of two tricycle robots as shown in Fig. 1.

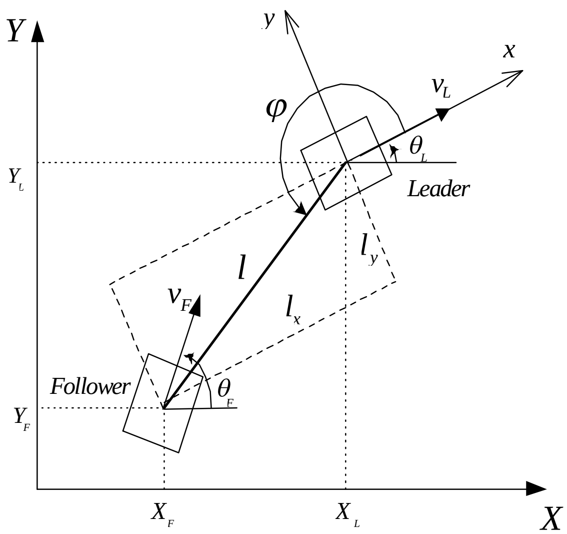

Fig.1. Leader-following configuration of two tricycle robots

In Fig. 1, X-Y is the world coordinates, and x-y is the Cartesian coordinates fixed on the leader's body. (X_L, Y_L) and (X_F, Y_F) are the global positions of the leader and follower, in which the subscripts 'L' and 'F' represent leader and follower, respectively. v_L and v_F are the leader's and follower's linear velocities; θ_L and θ_F are their orientation angles. l and φ are the follower's relative distance and relative angle with respect to the leader.

Given the leader's position and orientation, and as long as (l, φ) is known and fixed, the follower's position will be unique. To achieve the desired formation, we need to control $l \to l^d$ and $\varphi \to \varphi^d$, in which the superscript 'd' means desired.

Note that (l, φ) is actually a polar coordinate representation of the follower's relative position with respect to the leader if we view the center of the leader as the origin of the polar coordinate. Apparently, the kinematics model of the formation system can be represented using (l, φ). However, such representation will lead to certain singular points in the controller, i.e., the denominator in the controller formula may be zero at certain points [4, 5]. As we know, the representation of the kinematics model of a mobile robot can use polar coordinates or Cartesian coordinates. Therefore, in order to eliminate the possible singularity points, we use Cartesian representation instead of polar coordinates.

In order to describe the relative positions between the leader and follower in Cartesian coordinates, we need to project the relative distance along the x and y directions. Thus, we have l_x and l_y as

$$\begin{cases} l_x = -(X_L - X_F)\cos\theta_L - (Y_L - Y_F)\sin\theta_L & (1) \\ l_y = (X_L - X_F)\sin\theta_L - (Y_L - Y_F)\cos\theta_L & (2) \end{cases}$$

in which l_x and l_y are the follower's relative positions along the x and y directions, respectively.

In Cartesian coordinates x-y, given the leader's position, and as long as (l_x, l_y) is known and fixed, the follower's position will be unique. Thus, the follower's relative position with respect to the leader can be represented by (l_x, l_y).

To achieve and maintain the desired formation, we need to control $l_x \to l_x^d$ and $l_y \to l_y^d$, where l_x^d and l_y^d are the desired relative positions along the x and y directions, respectively. Since in normal cases the relative distance l^d rather than l_x^d and l_y^d is given, we need to project l^d along the x and y directions to get l_x^d and l_y^d.

We have

$$l_x^d = l^d \cos\varphi^d,\ l_y^d = l^d \sin\varphi^d \quad (3)$$

and

$$\dot{l}_x^d = \dot{l}^d \cos\varphi^d - l^d \dot{\varphi}^d \sin\varphi^d \quad (4)$$

$$\dot{l}_y^d = \dot{l}^d \sin\varphi^d + l^d \dot{\varphi}^d \cos\varphi^d \quad (5)$$

In certain applications of mobile robot formation control, the desired relative distance l^d between the leader and follower is required to be a constant, while the relative angle φ^d be a time variant. Suppose l^d to be a constant value l_0, i.e., $\dot{l}^d$=0. Then, equations (4) and (5) change into:

$$\dot{l}_x^d = -l_0 \dot{\varphi}^d \sin\varphi^d,\ \dot{l}_y^d = l_0 \dot{\varphi}^d \cos\varphi^d \quad (6)$$

From (1), we have

$$\begin{aligned} \dot{l}_x &= -(\dot{X}_L - \dot{X}_F)\cos\theta_L + (X_L - X_F)\dot{\theta}_L \sin\theta_L \\ &\quad - (\dot{Y}_L - \dot{Y}_F)\sin\theta_L - (Y_L - Y_F)\dot{\theta}_L \cos\theta_L \\ &= l_y \dot{\theta}_L + \dot{X}_F \cos\theta_L + \dot{Y}_F \sin\theta_L - \dot{X}_L \cos\theta_L - \dot{Y}_L \sin\theta_L \\ &= l_y \dot{\theta}_L + \dot{X}_F \cos\theta_L + \dot{Y}_F \sin\theta_L - v_L \end{aligned} \quad (7)$$

where v_L represents the leader's linear velocity.

Defining a new state variable to represent the difference of the orientation angles between the leader and follower as $e_\theta = \theta_F - \theta_L$

i.e., $\theta_L = \theta_F - e_\theta$. (8)

Substituting (8) into (7), we have

$$\begin{aligned} \dot{l}_x &= l_y \dot{\theta}_L + \dot{X}_F \cos(\theta_F - e_\theta) + \dot{Y}_F \sin(\theta_F - e_\theta) - v_L \\ &= l_y \dot{\theta}_L - v_L + \cos e_\theta (\dot{X}_F \cos\theta_F + \dot{Y}_F \sin\theta_F) \\ &\quad - \sin e_\theta (\dot{Y}_F \cos\theta_F - \dot{X}_F \sin\theta_F) \end{aligned} \quad (9)$$

Since the nonholonomic constraint of a mobile robot is

$$\dot{Y}_F \cos\theta_F - \dot{X}_F \sin\theta_F = 0 \quad (10)$$

With (10), (9) becomes

$$\dot{l}_x = l_y w_L + v_F \cos e_\theta - v_L \quad (11)$$

where $w_L = \dot{\theta}_L$ represents the angular velocity of the leader, and v_F represents the follower's linear velocity.

Similarly from (2) we have

$$\dot{l}_y = -l_x w_L + v_F \sin e_\theta \quad (12)$$

Thus, the whole system's kinematics model is

$$\begin{cases} \dot{l}_x = l_y w_L + v_F \cos e_\theta - v_L & (13) \\ \dot{l}_y = -l_x w_L + v_F \sin e_\theta & (14) \\ \dot{e}_\theta = w_F - w_L & (15) \end{cases}$$

where w_F and v_F are the follower's angular and linear velocities; w_L and v_L are the leader's angular and linear velocities, respectively. By the leader-following approach, w_L and v_L are given and might be constant values or time-varying functions; w_F and v_F are control inputs.

Actually, instead of doing the preceding triangular computation, the system model can be derived directly by relative kinematics analysis. Equations (13) and (14) present nothing but the follower's relative linear velocities along the *x* and *y* coordinates with respect to the leader.

The control task is to design control laws for w_F and v_F to make l_x and l_y achieve the desired value for maintaining the desired formation, and also to make e_θ stable.

3. Controller Design

In a leader-follower configuration, given the leader's position and once the follower's relative distance and angle with respect to the leader are known, the follower's position can be determined. To use the leader-following approach, we assume that the angular and linear velocities of the leader are known. In order to achieve and maintain the desired formation between the leader and follower, we only need to control the follower's angular and linear velocities to achieve the relative distance and angle between them as desired. Therefore, the leader-following based mobile robot formation control can be considered as an extension of the tracking problem of the nonholonomic mobile robot.

A few controllers have been presented in the literature [4, 5, 6, 13]. [4] and [5] present a controller based on input-output linearization, but it is just locally stable. A new controller using a sliding mode control approach, which is globally stable, is presented in [13]. However, the design procedure is complicated. In this section, by using the direct Lyapunov method, we derive a new globally stable controller in a simpler manner.

Define two error states as:

$e_x = l_x^d - l_x$ and $e_y = l_y^d - l_y$

Then

$$\dot{e}_x = \dot{l}_x^d - \dot{l}_x = \dot{l}^d \cos\varphi^d - l^d \dot{\varphi}^d \sin\varphi^d - w_L(l^d \sin\varphi^d - e_y) - v_F \cos e_\theta + v_L$$

Similarly, we have

$$\dot{e}_y = \dot{l}^d \sin\varphi^d + l^d \dot{\varphi}^d \cos\varphi^d + w_L l^d \cos\varphi^d - e_x w_L - v_F \sin e_\theta$$

Then, the error dynamics of the whole system is

$$\begin{cases} \dot{e}_x = w_L e_y - v_F \cos e_\theta + \dot{l}^d \cos\varphi^d - l^d \dot{\varphi}^d \sin\varphi^d \\ \qquad - w_L l^d \sin\varphi^d + v_L & (16) \\ \dot{e}_y = -w_L e_x - v_F \sin e_\theta + \dot{l}^d \sin\varphi^d + l^d \dot{\varphi}^d \cos\varphi^d \\ \qquad + w_L l^d \cos\varphi^d & (17) \\ \dot{e}_\theta = w_F - w_L & (18) \end{cases}$$

For simplicity, we define

$$f_1 = \dot{l}^d \cos\varphi^d - l^d \dot{\varphi}^d \sin\varphi^d - w_L l^d \sin\varphi^d + v_L \quad (19)$$

$$f_2 = \dot{l}^d \sin\varphi^d + l^d \dot{\varphi}^d \cos\varphi^d + w_L l^d \cos\varphi^d \quad (20)$$

Then the error dynamics can be written as

$$\begin{cases} \dot{e}_x = w_L e_y - v_F \cos e_\theta + f_1 & (21) \\ \dot{e}_y = -w_L e_x - v_F \sin e_\theta + f_2 & (22) \\ \dot{e}_\theta = w_F - w_L & (23) \end{cases}$$

If the desired distance between the leader and follower is constant, i.e., $\dot{l}^d = 0$, and let $l^d = l_0$, then

$$f_1 = -l^d \dot{\varphi}^d \sin\varphi^d - w_L l^d \sin\varphi^d + v_L \quad (24)$$

$$f_2 = l^d \dot{\varphi}^d \cos\varphi^d + w_L l^d \cos\varphi^d \quad (25)$$

We assume that the given l^d, φ^d, w_L and v_L all are sufficiently smooth. Because of the saturation of the robot motors, w_L, v_L and their first derivatives are assumed to be bounded. Clearly, from (19) and (20) we know f_1 and f_2 are bounded, smooth, and known functions. We also assume that all states are measurable by noiseless sensors.

We need to design control laws for w_F and v_F to make e_θ stable, and e_x and e_y asymptotically stable or at least bounded.

To use the direct Lyapunov method, we select a candidate Lyapunov function as:

$$V = \tfrac{1}{2}\ln(1 + e_x^2 + e_y^2) + \tfrac{1}{2} e_\theta^2 \quad (26)$$

Clearly $V \geq 0$, and

$$\dot{V} = \frac{e_x \dot{e}_x + e_y \dot{e}_y}{1 + e_x^2 + e_y^2} + e_\theta \dot{e}_\theta = \frac{1}{1 + e_x^2 + e_y^2}(-e_x v_F \cos e_\theta + e_x f_1 - e_y v_F \sin e_\theta + e_y f_2) + e_\theta(w_F - w_L) \quad (27)$$

For simplicity, we define

$$E = 1 + e_x^2 + e_y^2 \tag{28}$$

Obviously $0 < \frac{1}{E} \le 1$.

Using (28), (27) becomes

$$\dot{V} = -\tfrac{1}{E} v_F (e_x \cos e_\theta + e_y \sin e_\theta) + \tfrac{1}{E}(e_x f_1 + e_y f_2) + e_\theta (w_F - w_L) \tag{29}$$

Select

$$w_F = w_L - k_1 e_\theta + \alpha \tag{30}$$

in which k_1 is a positive design constant and α is a design parameter that needs to be investigated further.

Substituting (30) into (29), we have

$$\dot{V} = -\tfrac{1}{E} v_F (e_x \cos e_\theta + e_y \sin e_\theta) + \tfrac{1}{E}(e_x f_1 + e_y f_2) - k_1 e_\theta^2 + \alpha e_\theta \tag{32}$$

Select

$$v_F = k_3 (e_x \cos e_\theta - e_y \sin e_\theta) + \beta \tag{33}$$

in which k_3 is a positive design constant, and β is a positive design parameter that could be a constant or bounded function.

Substituting (33) into (32), we have

$$\begin{aligned}\dot{V} &= -\tfrac{1}{E} k_3 (e_x^2 \cos^2 e_\theta - e_y^2 \sin^2 e_\theta) - \tfrac{1}{E}\beta (e_x \cos e_\theta + e_y \sin e_\theta) + \tfrac{1}{E}(e_x f_1 + e_y f_2) - k_1 e_\theta^2 + \alpha e_\theta \\ &= -k_1 e_\theta^2 - \tfrac{1}{E} k_3 e_x^2 \cos^2 e_\theta - (-\tfrac{1}{E} k_3 \sin^2 e_\theta) e_y^2 + \alpha e_\theta \\ &\quad - \tfrac{1}{E}\beta (e_x \cos e_\theta + e_y \sin e_\theta) + \tfrac{1}{E}(e_x f_1 + e_y f_2)\end{aligned} \tag{34}$$

Furthermore, we select

$$\alpha = -\tfrac{1}{E} k_2 e_y^2 sign(e_\theta) \tag{35}$$

i.e., $w_F = w_L - k_1 e_\theta - \frac{1}{E} k_2 e_y^2 sign(e_\theta)$ (36)

where k_2 is a positive design constant, and $k_2 > k_3 > 0$.

Proposition: Considering the formation control problem of two tricycle mobile robots as shown in Fig.1, for any given bounded and sufficiently smooth leader's path, the proposed control laws in (33) and (36) can achieve and maintain the relative distance and angle between the follower and the leader as desired with bounded errors. The whole system is also stable.

Proof: From (35) we have

$$\alpha e_\theta = -\tfrac{1}{E} k_2 e_y^2 |e_\theta| \tag{37}$$

To test the stability of the whole system, substituting (33) and (36) into (34), we have

$$\begin{aligned}\dot{V} &= -k_1 e_\theta^2 - \tfrac{1}{E} k_3 e_x^2 \cos^2 e_\theta - \tfrac{1}{E} e_y^2 (k_2 |e_\theta| - k_3 \sin^2 e_\theta) \\ &\quad - \tfrac{1}{E}\beta (e_x \cos e_\theta + e_y \sin e_\theta) + \tfrac{1}{E}(e_x f_1 + e_y f_2)\end{aligned} \tag{38}$$

Note that $sin^2 e_\theta$ is an increasing function in $[0, \pi/2]$, it is not hard to see $|e_\theta| \ge \sin^2 e_\theta$ for any e_θ. Thus, if $k_2 > k_3 > 0$, we have $k_2 |e_\theta| \ge k_3 \sin^2 e_\theta$.

For simplicity, we define

$$\gamma^2 = k_2 |e_\theta| - k_3 \sin^2 e_\theta \tag{39}$$

Then (38) can be written as

$$\begin{aligned}\dot{V} &= -k_1 e_\theta^2 - \tfrac{1}{E} k_3 e_x^2 \cos^2 e_\theta - \tfrac{1}{E} e_y^2 \gamma^2 - \tfrac{1}{E}\beta (e_x \cos e_\theta \\ &\quad + e_y \sin e_\theta) + \tfrac{1}{E}(e_x f_1 + e_y f_2)\end{aligned} \tag{40}$$

Since $\left|\frac{1}{E} e_x\right| \le 1$, $\left|\frac{1}{E} e_y\right| \le 1$, we have

$\left|-\frac{1}{E}\beta (e_x \cos e_\theta + e_y \sin e_\theta)\right| \le 2|\beta|$.

And because $\left|\frac{1}{E} e_x f_1\right| \le |f_1|$ and $\left|\frac{1}{E} e_x f_2\right| \le |f_2|$

$$\dot{V} \le -k_1 e_\theta^2 - \tfrac{k_3}{E} e_x^2 \cos^2 e_\theta - \tfrac{1}{E} e_y^2 \gamma^2 + 2|\beta| + |f_1| + |f_2| \tag{41}$$

Because f_1, f_2 and β all are bounded, by the comparison principle, from (41) we know $V(t)$ is bounded. Thus, from (26), e_θ, e_x and e_y all are bounded. From (33) and (36), we know that the control inputs are also bounded. Hence, we come to the conclusion that the whole system is stable.

Furthermore, if we select

$$\beta = \frac{e_x f_1 + e_y f_2 + e_y^2}{e_x \cos e_\theta + e_y \sin e_\theta + \varepsilon} \tag{42}$$

with

$$\begin{cases}\varepsilon = 0, & \text{if } e_x \cos e_\theta + e_y \sin e_\theta \ne 0 \\ \varepsilon = 1, & \text{if } e_x \cos e_\theta + e_y \sin e_\theta = 0\end{cases}$$

Thus, if $e_x \cos e_\theta + e_y \sin e_\theta \ne 0$ always holds, (40) will become

$$\dot{V} \le -k_1 e_\theta^2 - \tfrac{k_3}{E} e_x^2 \cos^2 e_\theta - \tfrac{1}{E} e_y^2 (\gamma^2 + 1) \tag{43}$$

From the invariance principle [9], we know that for any initial condition, the error states (e_θ, e_x, e_y) will approach ($0, 0, 0$) because ($0, 0, 0$) is the only invariant set to make the equality in (43) hold.

However, note that when (e_θ, e_x, e_y) approaches ($0, 0, 0$), $e_x \cos e_\theta + e_y \sin e_\theta$ will go to zero, then (40) becomes

$$\dot{V} \le -k_1 e_\theta^2 - \tfrac{k_3}{E} e_x^2 \cos^2 e_\theta - \tfrac{1}{E} e_y^2 \gamma^2 + |f_1| + |f_2| \tag{44}$$

which means V will not keep strictly converging to the invariant set. That is, ($0, 0, 0$) may not be essentially achieved, and the error states (e_θ, e_x, e_y) will stay in a bounded neighborhood around ($0, 0, 0$).

Therefore, as a matter of fact, by the control laws in (33) and (36), the system will have bounded tracking errors.

Remark: Because of the *sign* function in the control law of (36), e_θ will not necessarily converge to zero eventually, and its signal may also have chattering, as shown in the following simulation results.

4. SIMULATION STUDIES

To verify the presented new model and controller, we simulate the motion of a team of two tricycle mobile robots as shown in Fig. 1. We regulate the relative distance between the two robots as a constant value ($l^d=2.0m$). The simulation results for several different cases are listed as in Fig. 2~9. Figures 3, 5, 7, and 9 represent the trajectory of leader and follower. In Fig. 2, 4, 6, and 8, the four plots from the top to bottom show the relative distance l_x, l_y, relative distance error, and e_θ, respectively. The selected design constants as k_1=10, k_2=3.9 and k_3=1.5, which satisfy the required conditions stated in section 3.

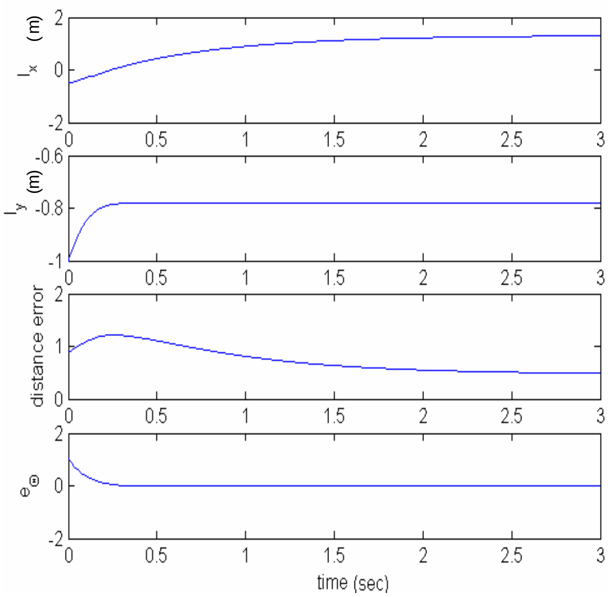

Fig. 2. Case 1: The leader goes along a straight line at a constant linear speed, and the follower keeps a constant relative angle and distance with respect to the leader.

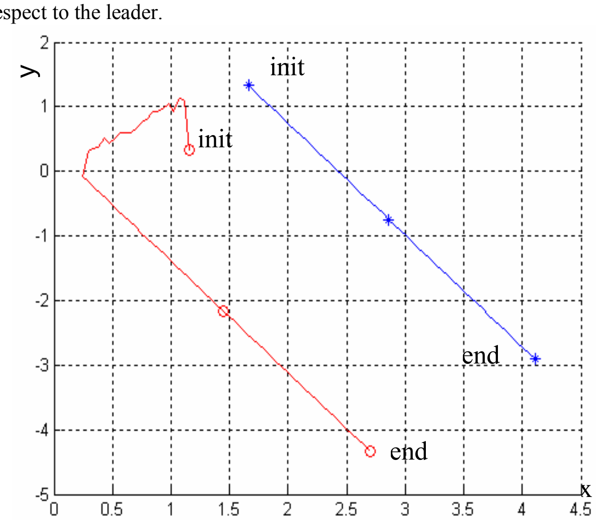

Fig.3. Trajectories of the leader and follower in Case 1 ('*' represents the leader and 'o' represents the follower; "ini" represents the initial position and "end" represents ending position).

Case 1: The leader robot goes along a straight line at a constant linear speed (v_L=0.15m/s) and the follower keeps a constant relative angle ($\varphi^d=7\pi/4$) with respect to the leader. The initial conditions for this case are l_x= -0.5m, l_y=-1.0m, and e_θ=π/3. Note that the relative distance error in Fig. 2 is bounded and e_θ approaches zero. Fig. 3 shows the trajectories of the leader and follower in this case.

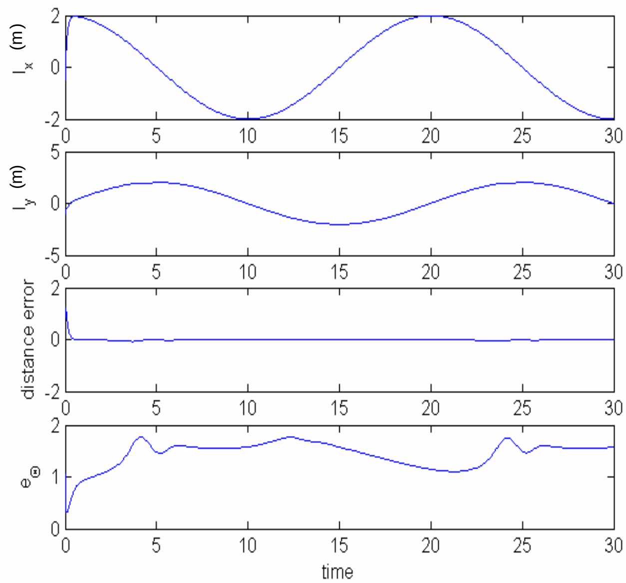

Fig. 4. Case 2: The leader goes along a straight line at a constant linear speed, and the follower keeps a constant relative distance and rotates around the leader at a constant relative angular speed.

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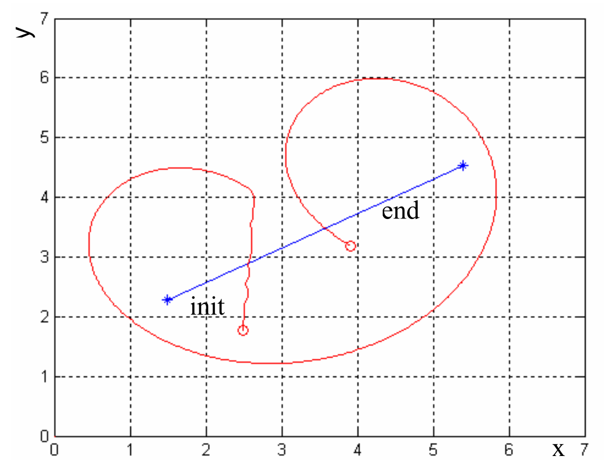

Fig.5. Trajectories of the leader and follower in Case 2 ('*' represents the leader and 'o' represents the follower; "ini" represents the initial position and "end" represents ending position).

Case 2: The leader goes along a straight line at a constant linear speed (v_L=0.15m/s), the follower rotates around the leader at a constant relative angular speed of 0.1π/s; the initial conditions for this case are l_x= 1.0m, l_y= -0.5m, and e_θ=π/3. Fig. 4 shows that the relative distance error is around zero and e_θ is bounded. The trajectories of leader and follower are shown in Fig. 5.

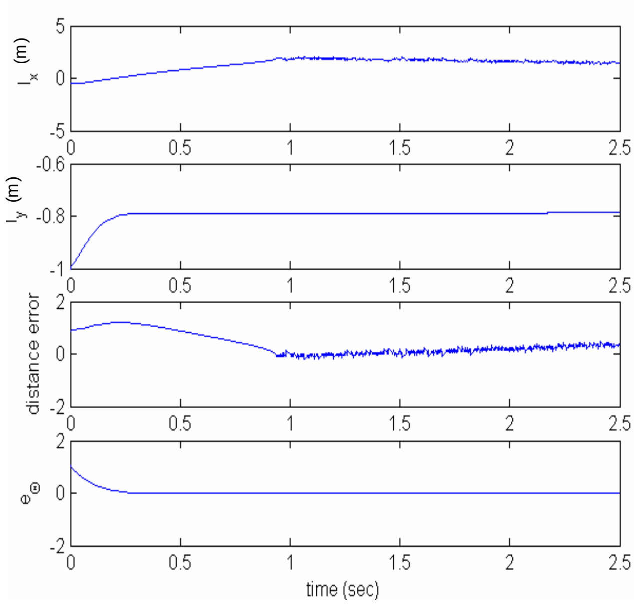

Fig. 6 Case 3: The leader moves at constant linear and angular speeds along a circle, and the follower keeps a constant relative angle with respect to the leader.

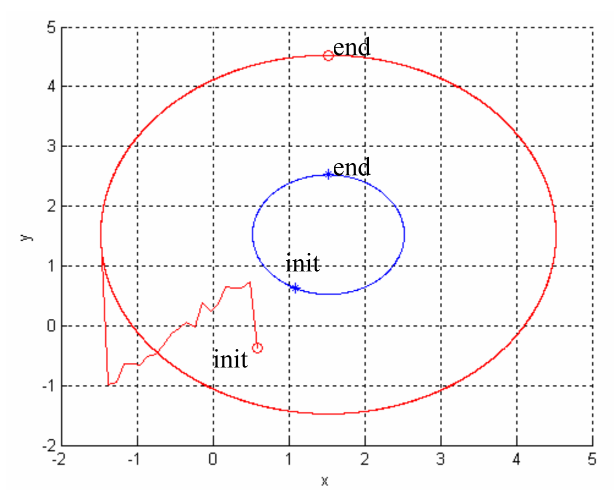

Fig. 7 Trajectories of the leader and follower in Case 3 ('*' represents the leader and 'o' represents the follower; "ini" represents the initial position and "end" represents ending position).

Case 3: The leader moves at a constant linear speed (v_L=0.10*m/s*) and a constant angular speed (0.1*π/s)* along a circle, and the follower robot keeps a constant relative angle ($\varphi^d=\pi/2$) with respect to the leader. The initial conditions for this case are l_x= -0.5m, l_y=-1.0m, and e_θ=π/3. Their trajectories are shown in Fig. 7. As we can see in Fig. 6, the relative distance error is bounded and e_θ approaches zero.

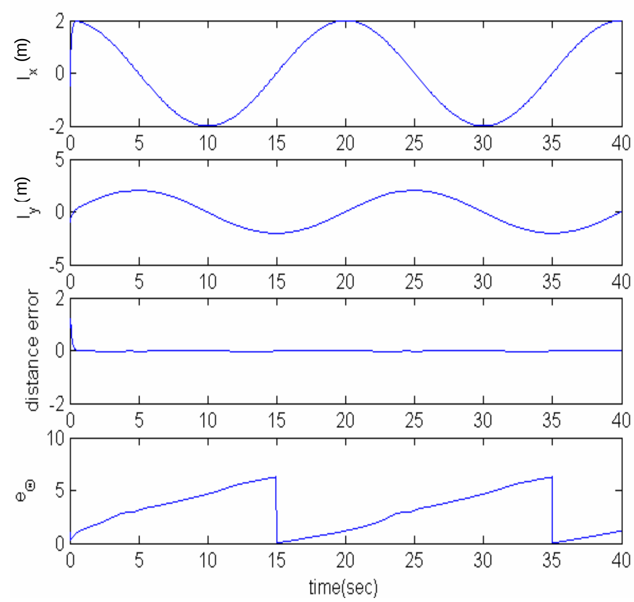

Fig. 8. Case 4: The leader moves at constant linear and angular speeds along a circle, and the follower rotates around the leader at a constant relative angular speed.

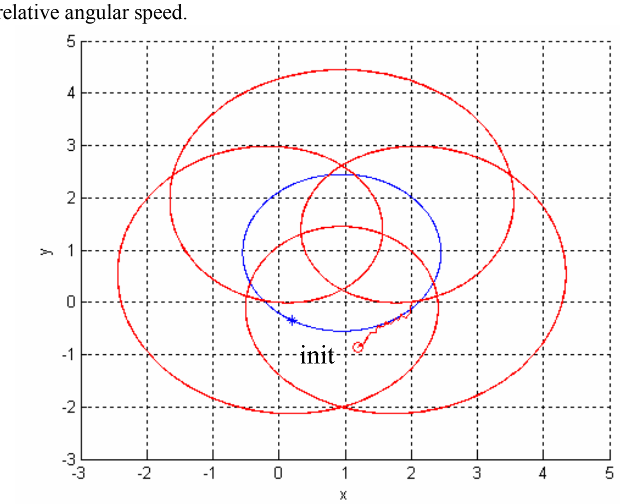

Fig. 9. Trajectories of the leader and follower in Case 4 ('*' represents the leader and 'o' represents the follower).

Case 4: The leader moves at a constant linear speed (v_L=0.15*m/s*) and a constant angular speed (0.1*π/s)* along a circle, and the follower rotates around the leader at a constant relative angular speed of 0.2*π/s.* The initial conditions for this case are l_x=1.0m, l_y= -0.5m, and e_θ=π/3. Their trajectories are shown in Fig. 9.

Unlike cases 1 and 3 (Figs. 2 and 6), e_θ doesn't converge to zero in cases 2 and 4 (Figs. 4 and 8). This is because that in cases 2 and 4, the follower rotates around the leader at a constant angular speed; and the relative angle between the leader and follower may keep changing. Thus, e_θ is bounded but does not necessarily converge to zero.

As we can see from the simulation results, the proposed controller can achieve the desired formation with bounded distance error and heading angle, and the whole system is stable.

5. CONCLUSION AND FUTURE WORK

In this paper, we present a new kinematics model for the leader-following based formation control of mobile robots by using Cartesian coordinates. Based on this new model, we use the Lyapunov direct method to derive a globally stable controller without singular points. The effectiveness of the controller is verified by simulation results.

The future research work will focus on extending the preliminary results to more general applications by employing more than two robots in formation control and taking into consideration of robot dynamics, model uncertainties, and noise.

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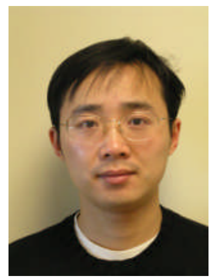

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